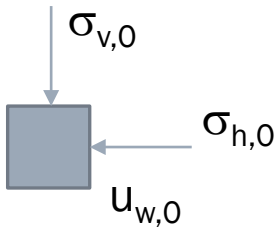
Geomechanics

LECTURE 11



IN-SITU STRESS

DR. ALESSIO FERRARI

Laboratory of soil mechanics – Fall 2025

Access the QUIZ

https://etc.ch/KRsz

Basic concepts

Elasticity - Linear & Non-linear

Plasticity Perfect-plasticity

Hardening elasto-plasticity

Critical state concept

Classical constitutive framework

Modified Cam-Clay

Practical aspects

Numerical modelling

Geomechanics in Practice

In-situ stress

Course Project

Retaining structures

Advanced constitutive models

Hydro-mechanical coupling

Unsaturated behaviour

Time-dependent behaviour

Topics

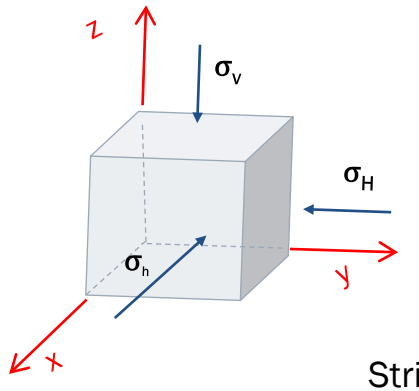
Content

- Introduction
- In-situ stress states
- Determination of the in-situ stress

Introduction

Terminology – Mathematical description

A simple and common assumption



$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma_h & 0 & 0 \\ 0 & \sigma_H & 0 \\ 0 & 0 & \sigma_v \end{pmatrix}$$

Strictly valid if horizontal topographic surface in a homogeneous medium, at most according to horizontal layers

State of stress in a point written in the principal stress system assuming the **vertical direction as principal one**

4 unknowns: the **magnitude** of the three principal stresses and the **direction** of one of the two horizontal principal stresses

The lateral stress coefficient

Coefficient of lateral stress

The **ratio** of one horizontal stress with respect to the vertical effective stress

$$K = \frac{\sigma'_i}{\sigma'_v}$$

i: generic horizontal stress

Coefficient of lateral stress at rest

The ratio of one horizontal stress with respect to the vertical stress in the case of **zero horizontal strains**

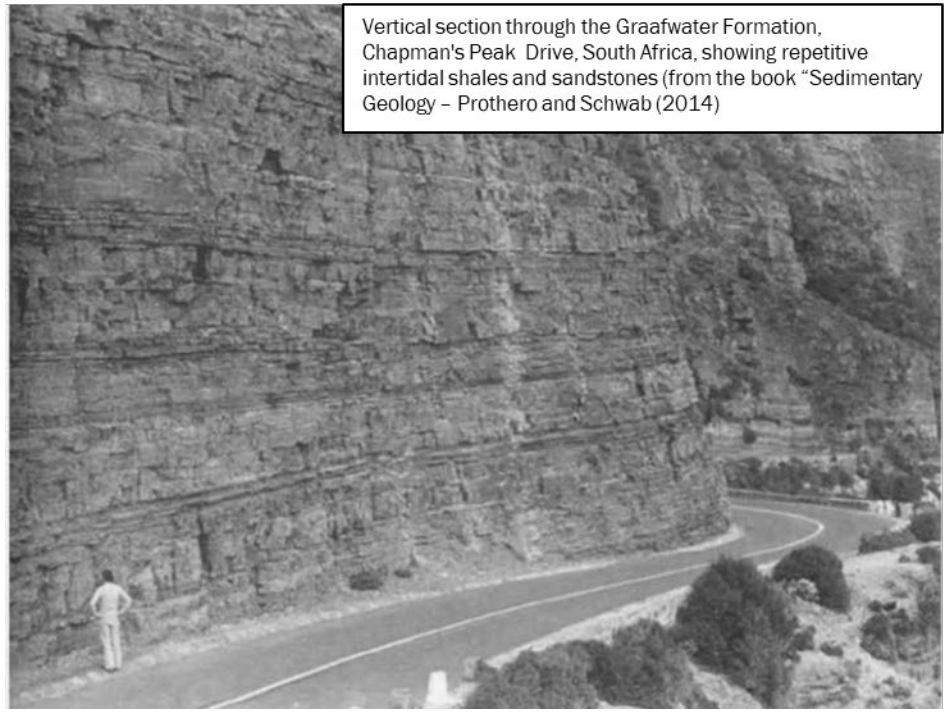
$$\varepsilon_i = 0 \rightarrow K_0 = \frac{\sigma'_i}{\sigma'_v}$$

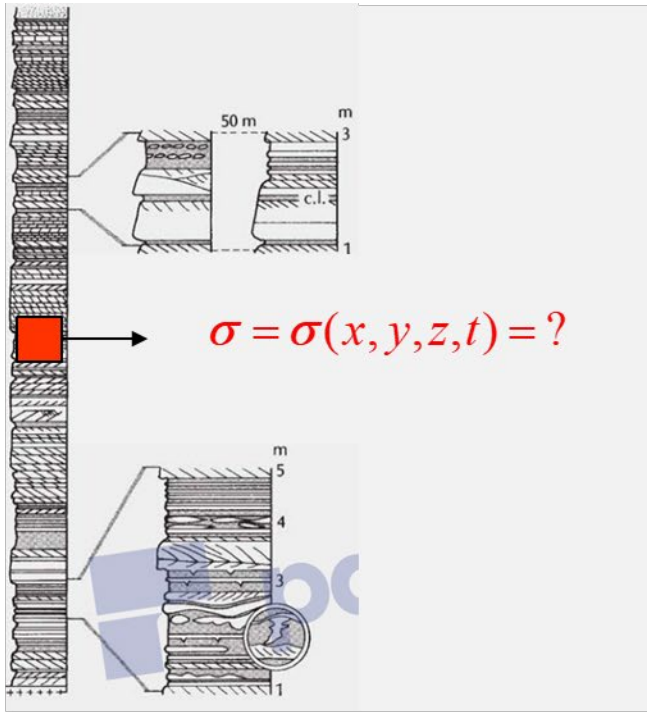
i: generic horizontal stress

Determination of the in-situ vertical stress, σ_v

$$\sigma_v = \int_0^z \rho(z) g dz \cong \bar{\rho} g z$$

$\rho(z)$: density as a function of depth z
g: gravitational acceleration constant
$\bar{\rho}$: mean density of the rocks above the depth z

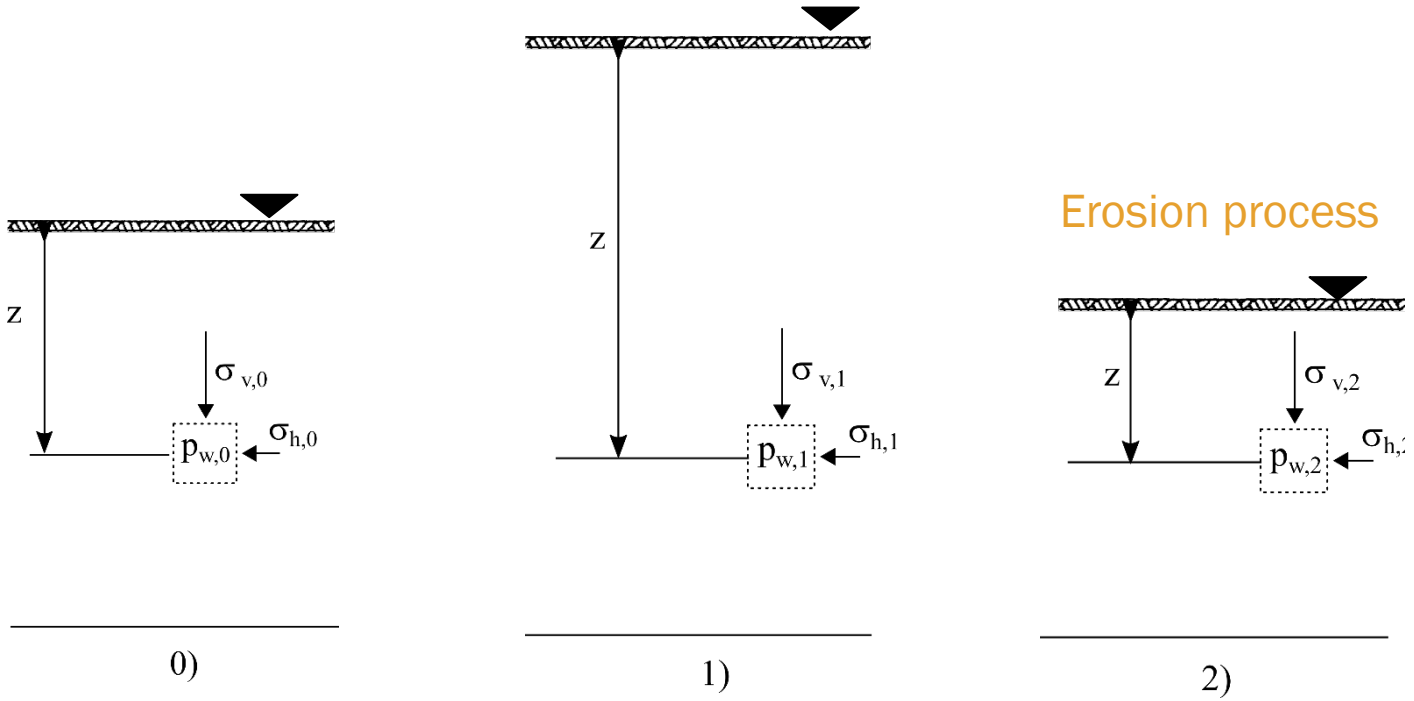

Vertical section through the Graafwater Formation, Chapman's Peak Drive, South Africa, showing repetitive intertidal shales and sandstones (from the book "Sedimentary Geology – Prothero and Schwab (2014)



- **Average densities** are required
- For computing the **vertical effective stress** it is necessary to know the **water pressure** at the depth of interest (**hydrostatic or not**)

The variation of the vertical effective stress with history

Sedimentation process



Erosion process

The **vertical effective stress** at each time depends only on the loads and the water pressure currently existing

$$\sigma'_v = \sigma_v - p_w$$

Example:

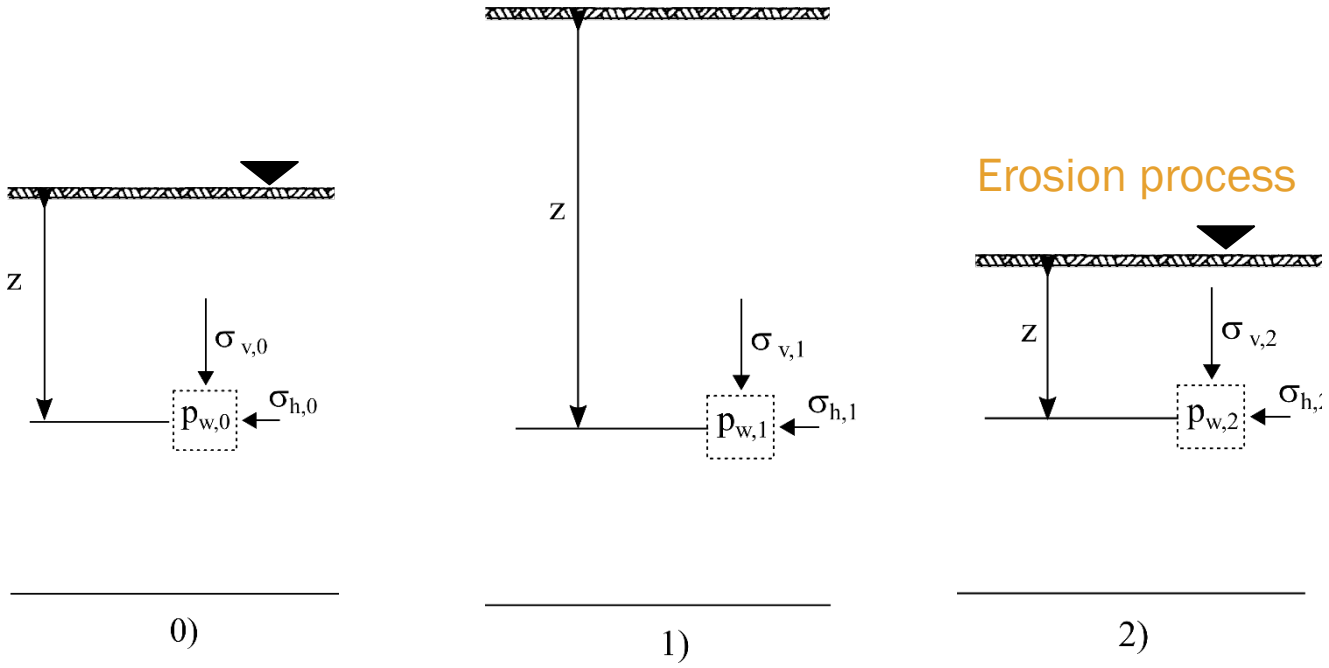
$z = 5\ m$
$\gamma_{sat} = 1.9 Mg/m^3$
$\gamma' = 0.9\ Mg/m^3$
$\sigma'_v = 44.1\ kN/m^2$

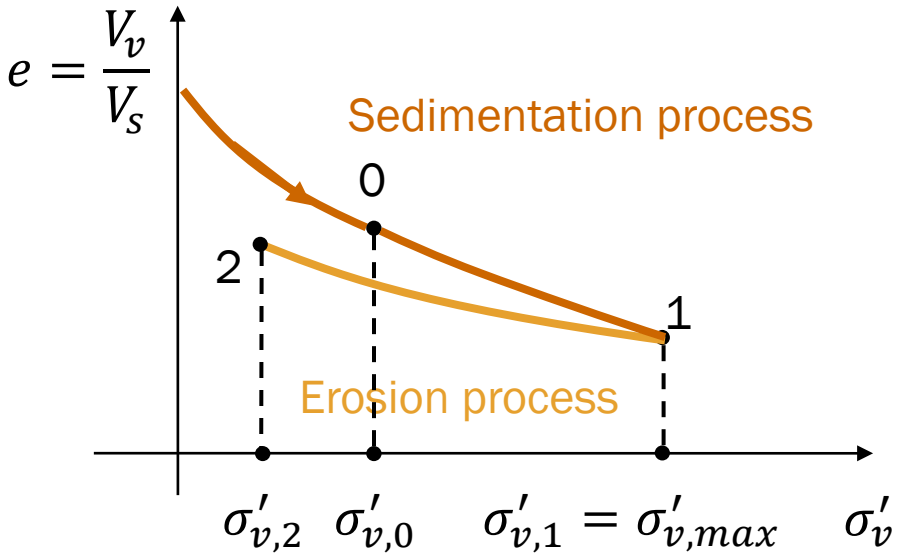
$z = 10\ m$
$\gamma_{sat} = 1.9 Mg/m^3$
$\gamma' = 0.9\ Mg/m^3$
$\sigma'_v = 88.3\ kN/m^2$

$z = 3\ m$
$\gamma_{sat} = 1.9 Mg/m^3$
$\gamma' = 0.9\ Mg/m^3$
$\sigma'_v = 26.5\ kN/m^2$

The variation of the vertical effective stress with history

Sedimentation process





The overconsolidatio ratio (OCR)

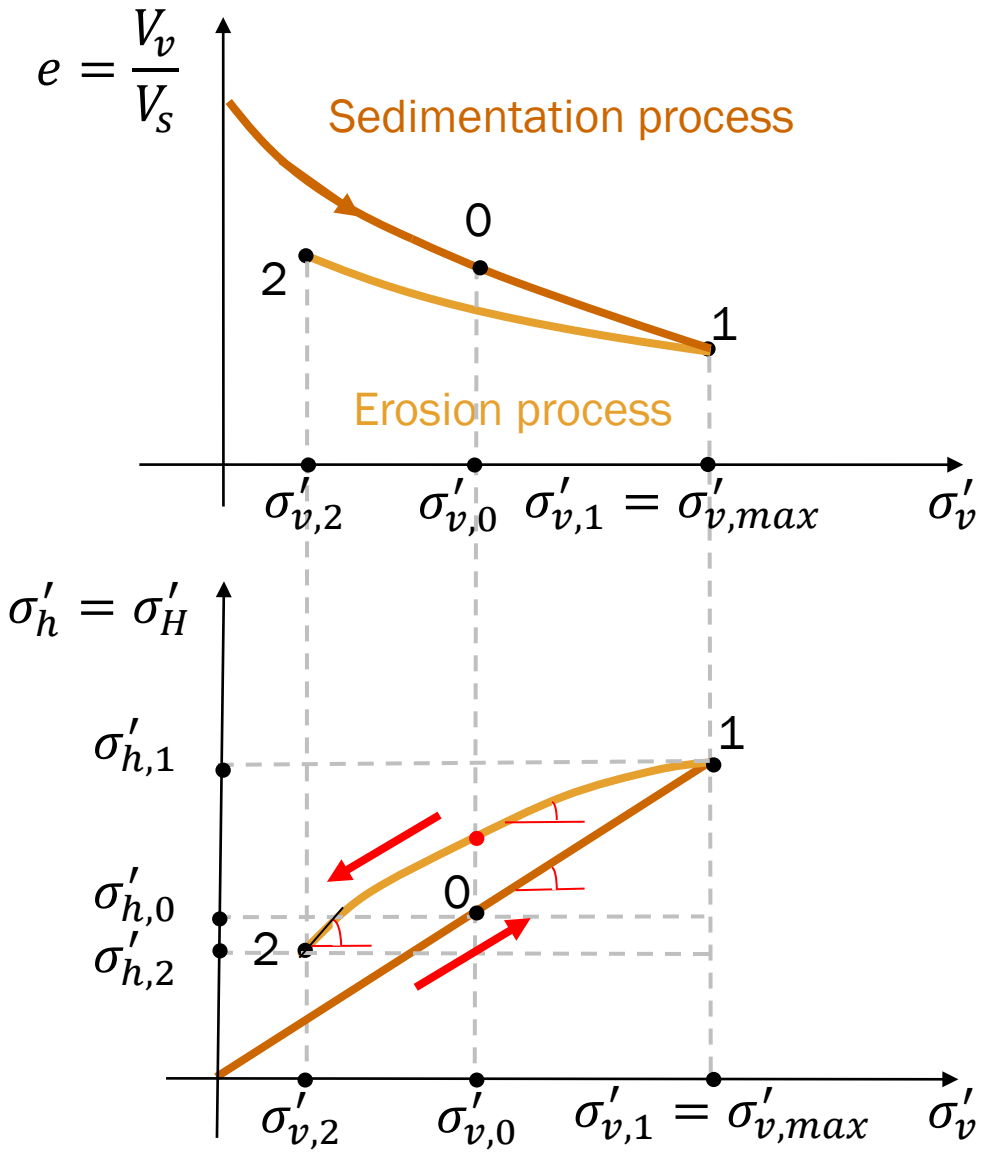
$$OCR_{1D} = \frac{\sigma'_{v,max}}{\sigma'_{v,actual}}$$

$0 \rightarrow 1$ $\quad OCR_{1D} = 1$ normally consolidated (NC)

$1 \rightarrow 2$ $\quad OCR_{1D} > 1$ overconsolidated (OC)

These types of processes generally involve very **large areas,** and the **horizontal strains** are **considered negligible.**

The variation of the horizontal effective stress with history



LOADING - UNLOADING

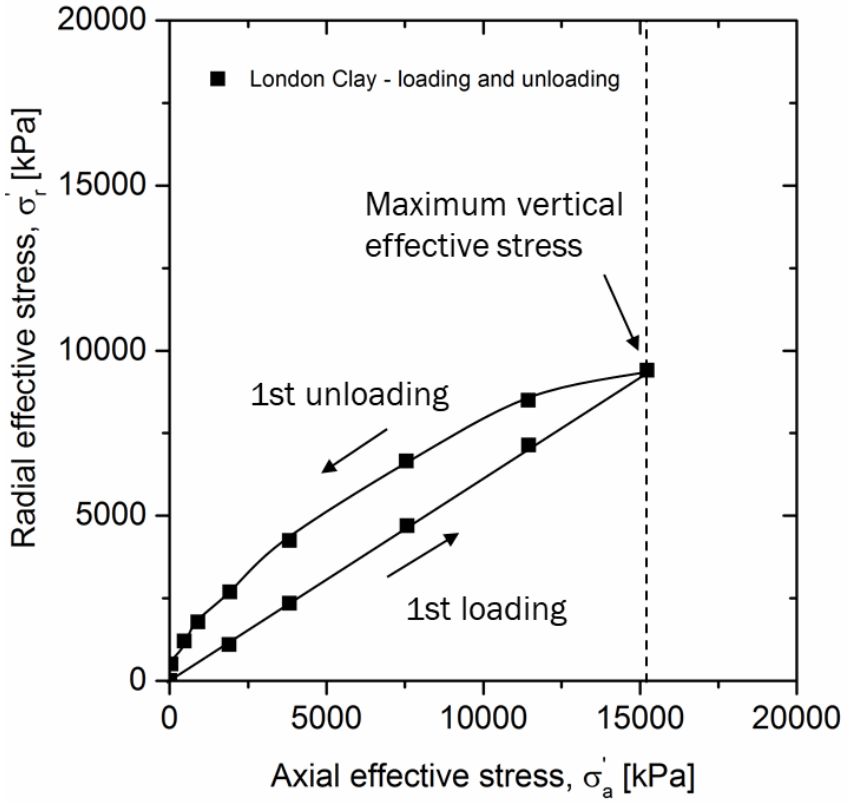
- As the vertical effective stress increases/decreases, the horizontal effective stress increases/decreases
- For a given vertical effective stress, there may be different values of horizontal stress (stress history dependency)
- K_0 is constant if the geomaterial is NC
- K_0 varies if the geomaterial is OC

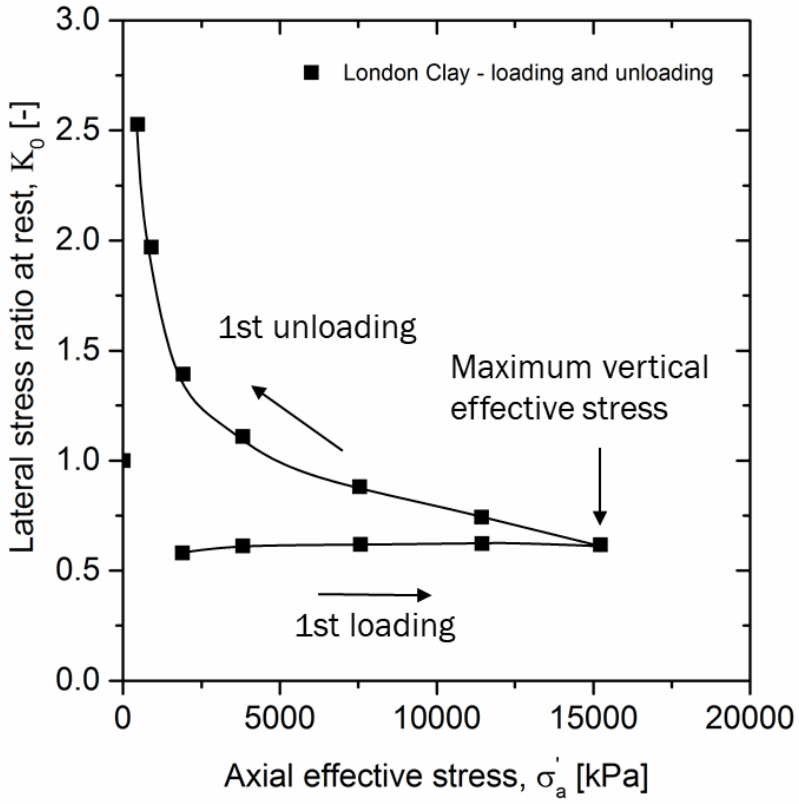
Other mechanisms causing overconsolidation are: exacavation, rise of the groundwater table, removal of surcharge loads…

The variation of the horizontal effective stress with history

Example of experimental data (Brooker and Ireland, 1965)

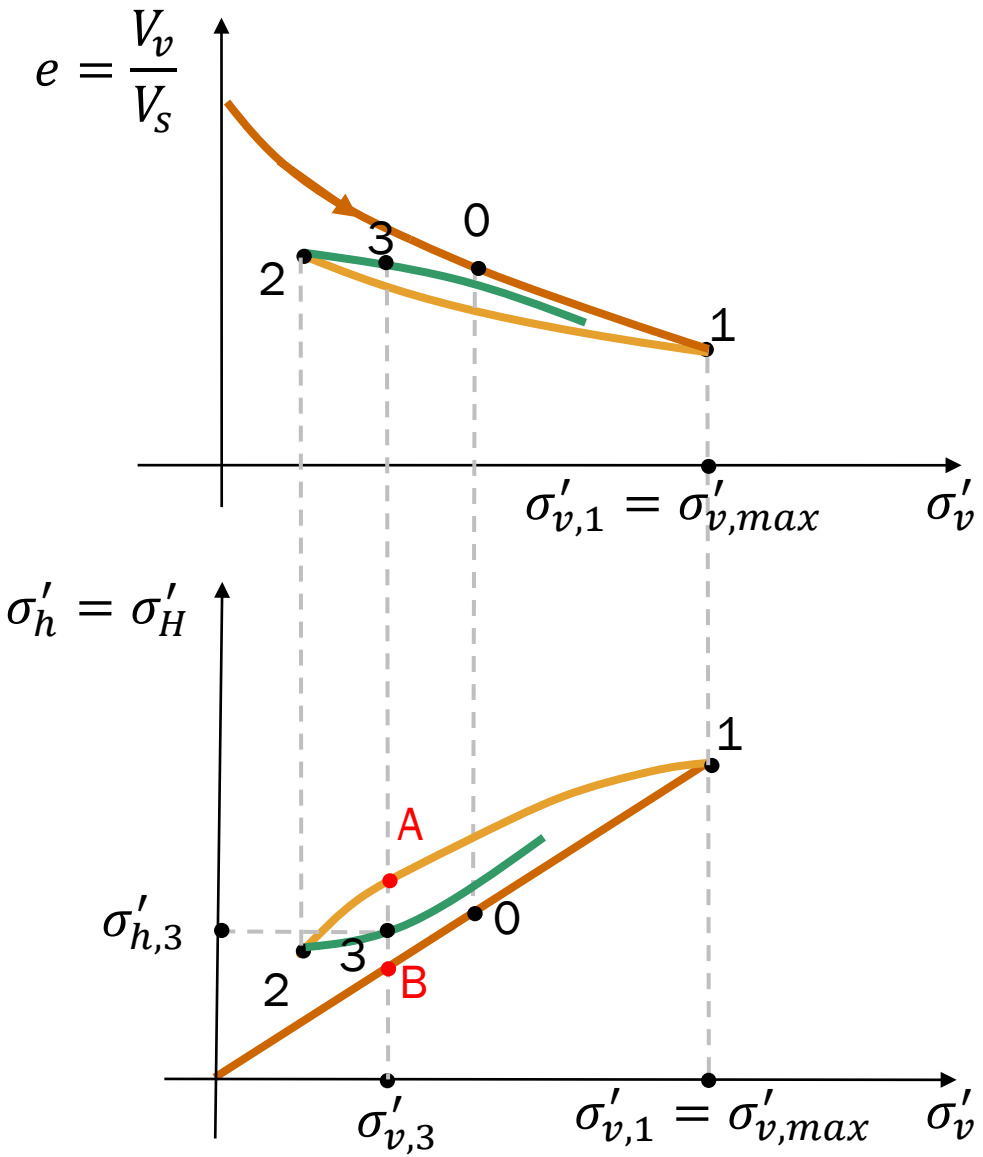
LONDON CLAY (I_p=38.4 %, clay fraction=64 %, c'=1.9, phi'=17.5°) – Oedometer tests





- The trend of the horizontal stress is exactly like the one reported in the previous slide
- During the first loading phase the K_0 remains practically constant
- During the unloading phase (i.e. geomaterial OC) K_0 increases and can reach values much larger than 1.

The variation of the horizontal effective stress with history



LOADING – UNLOADING - **RELOADING**

- For a given vertical effective stress, the horizontal effective stress is different depending if we are:
  - in the 1st loading
  - in the 1st unloading
  - in the 1st reloading

- For a given OCR, there can be a different horizontal effective stress (points A and 3)

$$OCR_{1D}(\text{A}) = \frac{\sigma'_{v,max}}{\sigma'_v(A)} \qquad OCR_{1D}(3) = \frac{\sigma'_{v,max}}{\sigma'_v(3)}$$

$$OCR_{1D}(\text{A}) = OCR_{1D}(3)$$

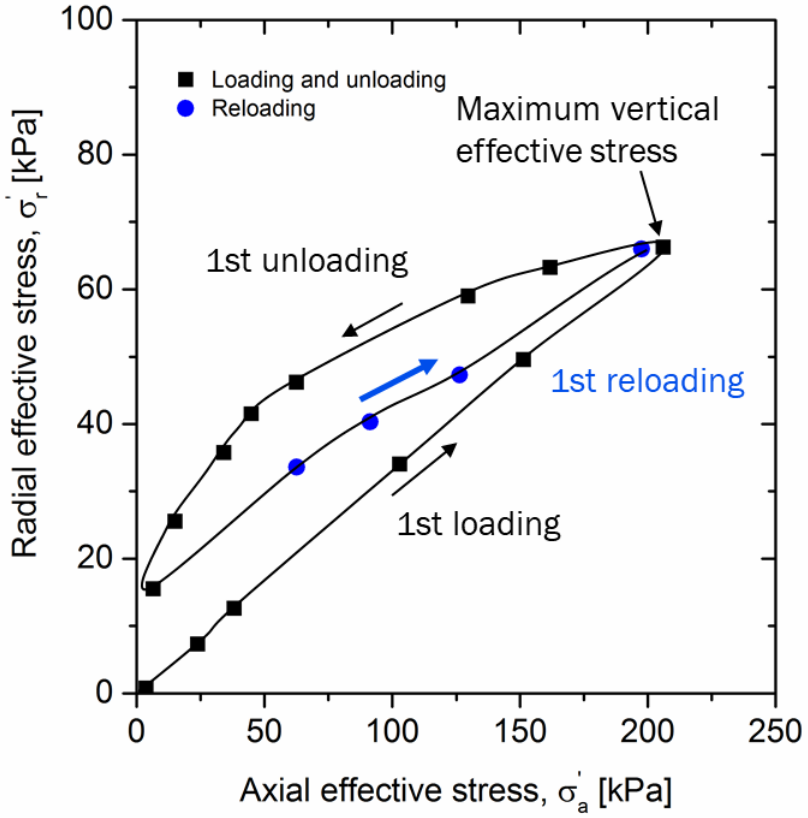
- Subsequent loading-reloading will follow paths which depends also on the eventual new maximum vertical effective stress reached

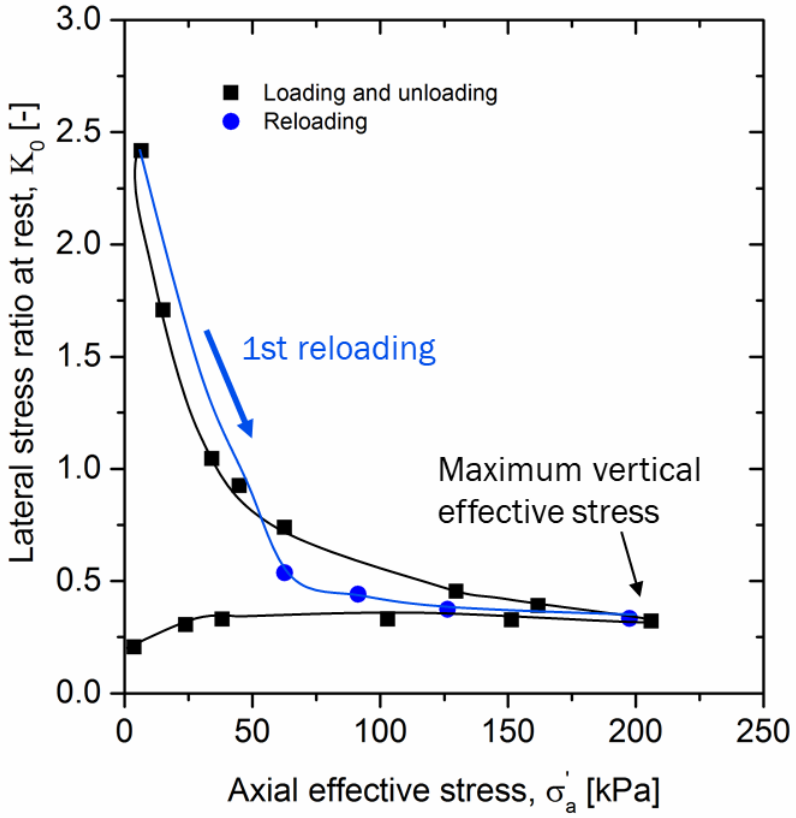
The variation of the horizontal effective stress with history

Example of experimental data (Mayne and Kulhawy, 1982)

Sand – Oedometer tests

LOADING – UNLOADING - **RELOADING**





- The trend of the horizontal stress is like the one reported in the previous slide
- During the first reloading phase (i.e. OC geomaterial) the K_0 changes (although to a lesser extent than in the case of 1st loading)

Key aspects

- In order to completely define the stress state we need all the components of the **stress tensor**, in the simplest case the problem is reduced to the determination of the vertical effective stress and of the horizontal effective stress.

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}(x,y,z,t) = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix} \quad \Rightarrow \quad \boldsymbol{\sigma} = \begin{pmatrix} \sigma_h & 0 & 0 \\ 0 & \sigma_H & 0 \\ 0 & 0 & \sigma_v \end{pmatrix}$$

The effective values are computed by knowing the pore water pressure distribution at the site of interest.

- The horizontal stress can be estimated by knowing the vertical effective stress (no stress history dependency) and the K_0.

$$\sigma'_v \quad \text{and} \quad K_0 = \frac{\sigma'_h}{\sigma'_v} \quad \Rightarrow \quad \sigma'_h = K_0 \sigma'_v$$

- The K_0 depends on several factors. For a given geomaterial (e.g. shear strength and plastic characteristics), it is a function of the stress history.

Determination of the in-situ stress

The estimation of the lateral stress coefficient

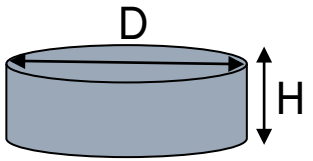
How to determine the K_0 for your problem? Two different options:

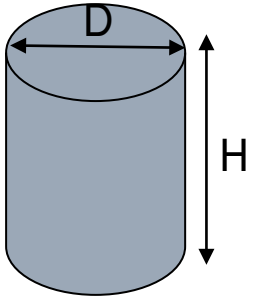
1. Perform K_0 tests on your geomaterial(s)
2. Use empirical relationship developed thanks to the existing experimental database

1. Perform K0 tests on your geomaterial(s)

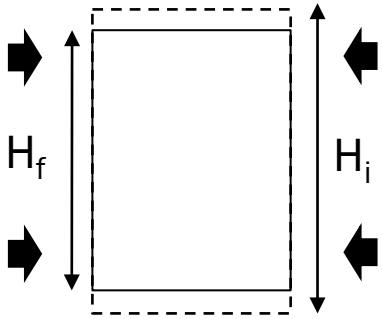
<u>Type of laboratory apparatus</u>

- A special oedometer cell which allows the radial stress measurements or
- A triaxial cell





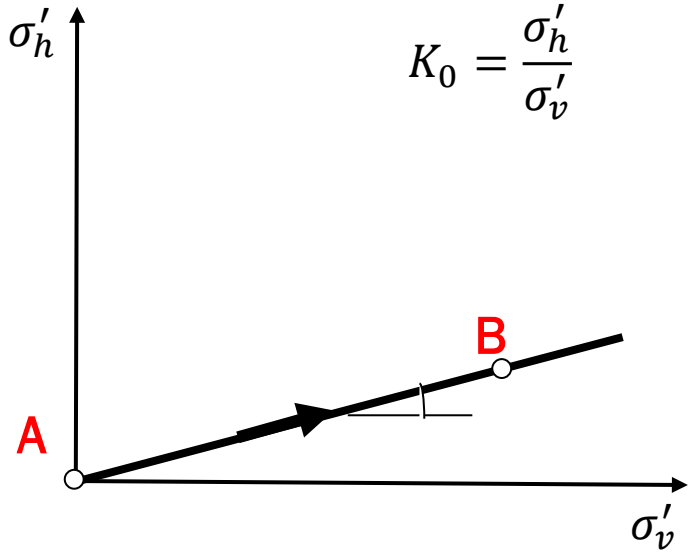
<u>Type of strain</u>

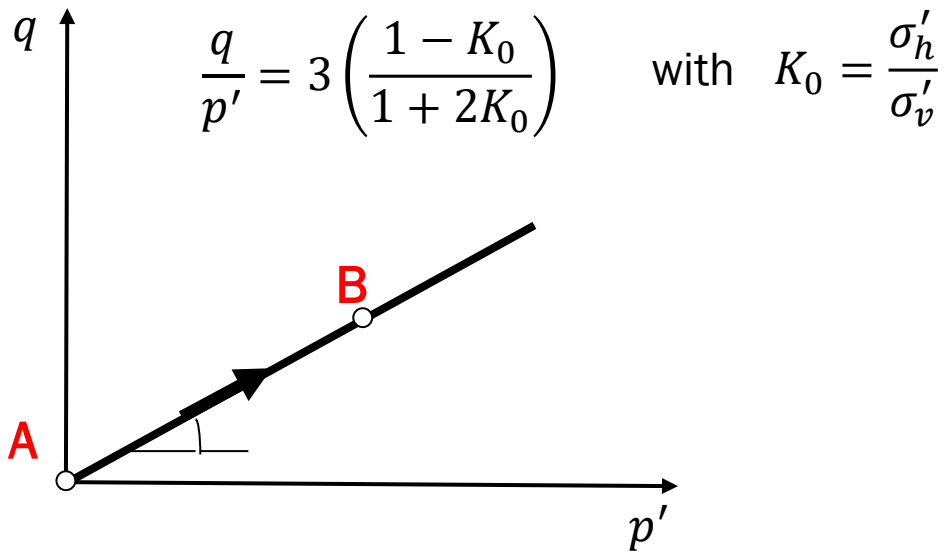


- Axial effective stress is imposed (the actual in-situ vertical effective stress or eventuallly more complex stress history are reproduced)
- Horizontal strains are prevented
- Radial stress is measured

K0-test

Type of stress path in different planes





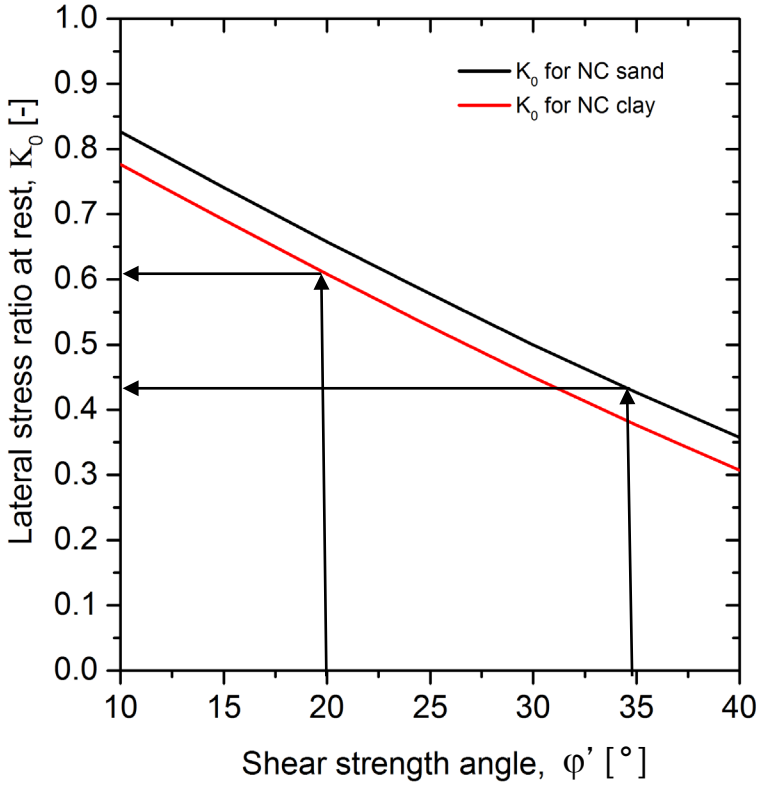
The results can be plotted in the plane you prefer (simply you will get different slopes depending on the variables you choose) but in practice you can estimate K_0 for the vertical effective stress of interest (or directly the horizontal stress).

Empirical relations

2. Use empirical relationship developed thanks to the existing experimental database

Several K_0 tests have been performed in the past, therefore different formulas have been provided relating the K_0 to other parameters

NC geomaterials (i.e. they have never been subjected to a higher effective stress than the current one)



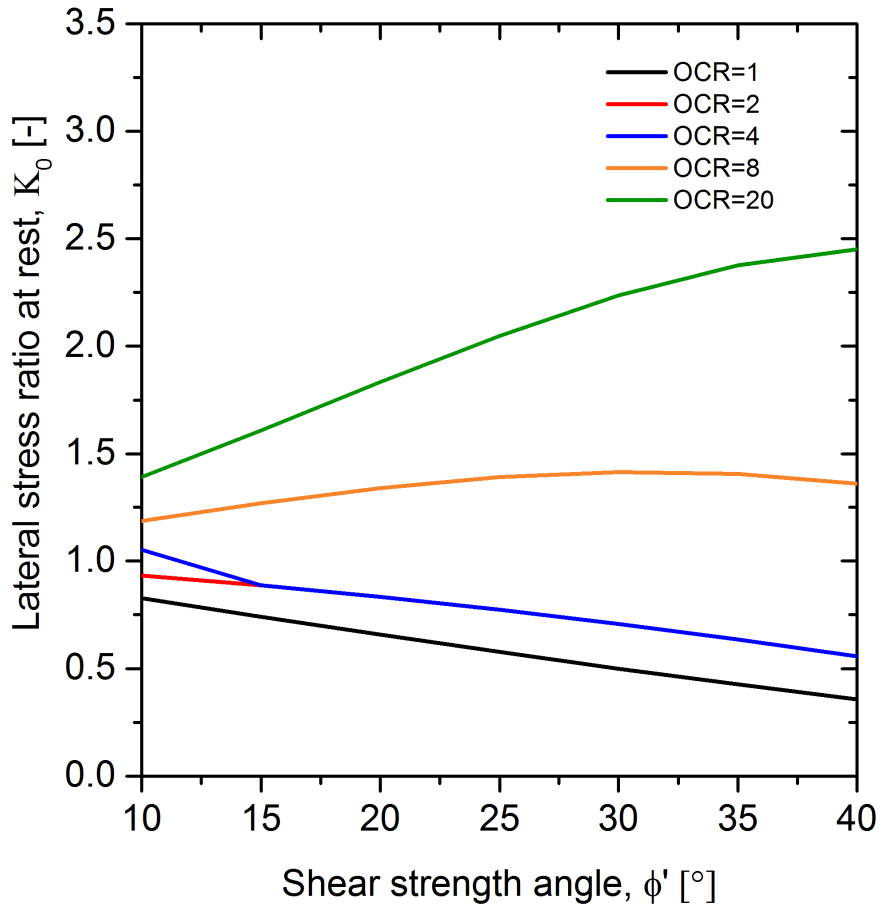
$K_0 \cong 1 - sin\varphi'$ For NC sand (Jaky, 1944)

$K_0 \cong 0.95 - sin\varphi'$ For NC clay (Brooker and Ireland, 1965)

The shear strength angle needs to be known

Empirical relations

OC geomaterials (i.e. they have been subjected to a higher effective stress than the current one)



For a simple loading unloading process (statistical study on 170 different soils)

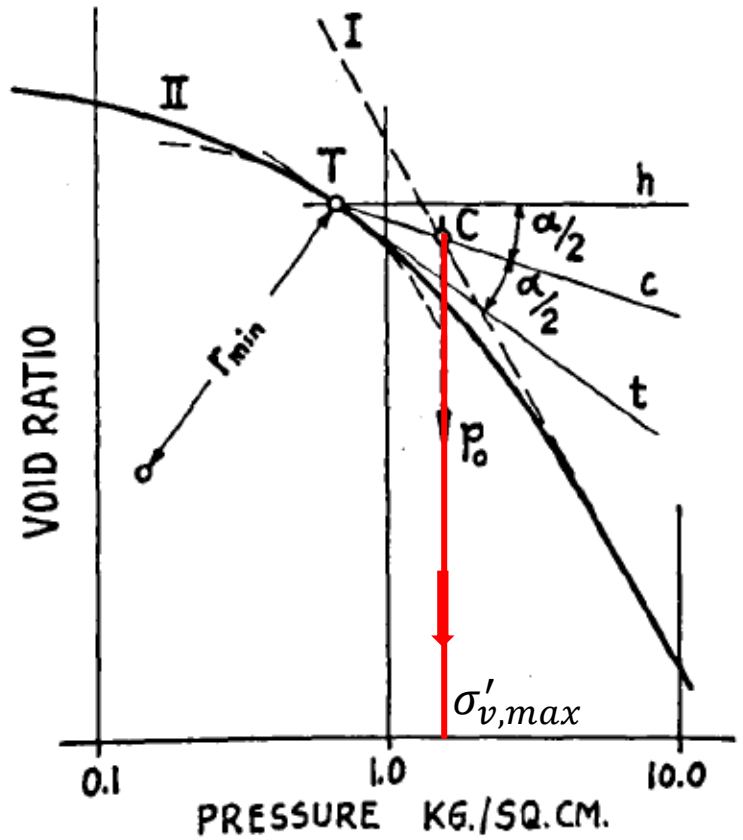
$$K_0 = (1 - sin\varphi')OCR^{sin\varphi'}$$

Mayne and Kulhawy (1982)

The shear strength angle and the OCR need to be known

Empirical relations

Determination of the OCR – Casagrande's method



$$OCR_{1D} = \frac{\sigma'_{v,max}}{\sigma'_{v,actual}}$$

Oedometer tests

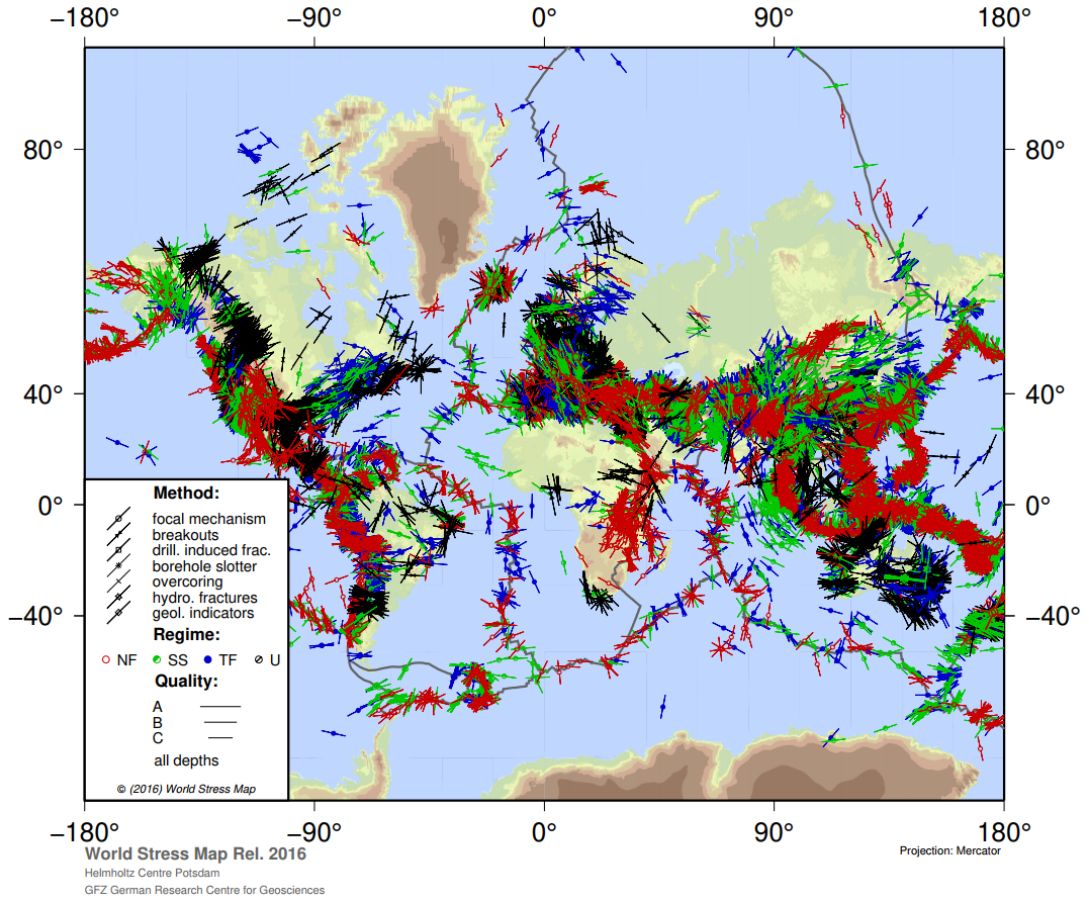
Post processing stages:

a. Identification of the maximum point of curvature of the experimental curve in the plane $e - \sigma'_v$ (i.e. point T).
b. The horizontal (h) and the tangent (t) to the curve passing through point T are traced.
c. The bisector of the angle between t and h is determined.
d. The intersection between the straight line of the experimental curve and the bisector is determined.

Casagrande, 1936. The determination of the pre-consolidation load and its practical significance.

The World Stress Map (WSP)

- It collects existing information of the stress state of the Earth's crust (42,870 data records within the upper 40 km of the Earth's crust).
- Data are classified according to the method used to estimate them and a quality index is associated with each information.
- It is useful for a global and regional analysis of the stress patterns.



Useful information

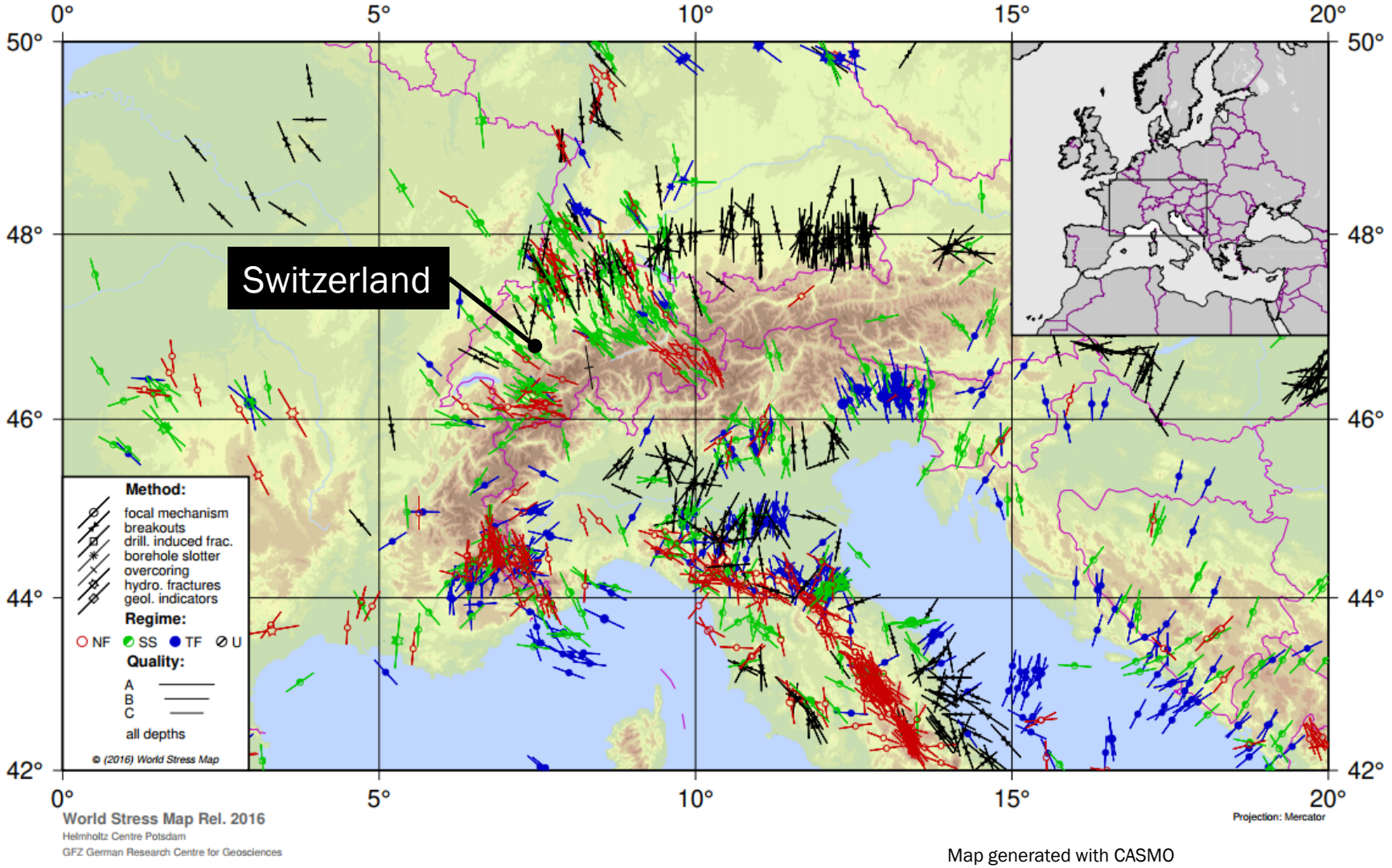
- The orientation of the maximum horizontal stress component.
- The tectonic regime deduced from the stress information.

Example of applications

Reservoirs, tunnel, boreholes, waste disposal sites

Map generated with CASMO (http://www.world-stress-map.org/casmo/)
Heidbach, Oliver; Rajabi, Mojtaba; Reiter, Karsten; Ziegler, Moritz; WSM Team (2016):
World Stress Map Database Release 2016. V. 1.1.
GFZ Data Services. http://doi.org/10.5880/WSM.2016.001

The World Stress Map (WSP)



261 data records in Switzerland
between 0 and 30 km depth

Map generated with CASMO
(http://www.world-stress-map.org/casmo/)
Heidbach, Oliver; Rajabi, Mojtaba; Reiter, Karsten; Ziegler, Moritz; WSM Team (2016):
World Stress Map Database Release 2016. V. 1.1.
GFZ Data Services. http://doi.org/10.5880/WSM.2016.001

Key aspects

1. Knowing the stress state at one point means knowing the components of a **second order tensor**.

2. The initial stress state is the **first point of a generic stress path**; it affects the **stiffness and strength** of the geomaterial.

3. Under the common assumption of verticality and horizontality of the principal stresses, only the vertical effective stress and the **K_0** must be estimated.

4. The coefficient of lateral stress is a function of several factors and depends not only on the loads currently acting but also on the **loads that were acting in the past**. It is important to know if the geomaterial is normal-consolidated or overconsolidated.

5. K_0 can be estimated by asking for **K_0-laboratory tests** or by using **formulas of empirical derivation**.

6. There is a **database** available on the website http://www.world-stress-map.org/casmo in which some information concerning the orientation of the principal stresses are collected and available for consultation.

Thank you for your attention